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THREE-DIMENSIONAL FLOW OF A HYPERSONIC DUSTY GAS OVER A WING

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The atmosphere always contains an impurity of fine solid particles (dust) in some concentration. This generates interest in investigating what happens when bodies fly through clouds of such particles, and also the possible change of aerodynamic characteristics. By using approximate theories of hypersonic flow [1] and the usual simplifications of the theory of two-phase flow [2], we can investigate the problem analytically.

A number of papers (e.g., [3, 4]) have examined hypersonic flow of a dusty gas over bodies of simple shape under the assumption that the presence of the impurity does not influence the gas flow. Flow over a thin wedge allowing for the mutual influence of the phases was investigated in [5].

In passing through the bow shock the gas parameters change sharply, but the parameters of the impurity particles remain continuous [5, 6]. According to the degree of accommodation of their velocity and temperature to the corresponding values of the carrier phase, we distinguish two limiting regimes of two-phase flow: "frozen," when the accommodation proceeds only slowly and the changes of particle parameters are negligible, and "equilibrium," where accommodation proceeds very rapidly in a narrow relaxation zone near the shock [6], and the phase parameters are the same in the main part of the field.

The present paper uses the thin-shock-layer method [1, 7] to study three-dimensional hypersonic flow over a short wing at finite angle of attack in the intermediate regime when the relaxation zone occupies the entire shock layer adjoining the windward surface of the wing. The particle velocity, temperature, and concentration change markedly across the shock layer. However, in reality, because of the high gas density in the layer, the influence of

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the impurity of the gas-dynamic variables in the main (Newtonian) approximation is negligible, but it must be accounted for in the successive approximations, particularly when adjusting the Newtonian formula for pressure.

1. We consider three-dimensional hypersonic flow of a dusty gas over a wing, assuming that the motion of the gas and the dust is described by the equations of flow of a continuous medium. We postulate that all the particles of the impurity are identical, are spherical in shape, and do not change during the motion. We shall neglect collisions of particles and their Brownian motion, and also the volume concentration $\tau \ll 1$. The gas viscosity is accounted for only when there is interphase interaction.

As usual, we denote by $V = (u, v, w)$, p, ρ, and T, respectively, the velocity vector, the pressure, density, and temperature. Functions referring to particles have the subscript p. We write the system of equations describing stationary flow of the two-phase medium investigated in the form

$$\begin{gathered}\nabla\cdot(\rho\mathbf{V}) = 0,\ \rho(\mathbf{V}\cdot\nabla)\mathbf{V} = -\nabla p + N\mathbf{f},\quad \rho c_V\mathbf{V}\cdot\nabla T = p\mathbf{V}\cdot\nabla\ln\rho + \\ + N\mathbf{f}\cdot(\mathbf{V}_p - \mathbf{V}) - NQ,\ p = c_V(\varkappa - 1)\rho T,\ \nabla\cdot(N\mathbf{V}_p) = 0,\ m_p(\mathbf{V}_p\cdot\nabla)\mathbf{V}_p = \\ = -\mathbf{f},\ m_p c\mathbf{V}_p\cdot\nabla T_p = Q,\ \mathbf{f} = \frac{1}{2}c_D\pi a^2\rho q(\mathbf{V}_p - \mathbf{V}),\quad Q = \sigma(T - T_p), \\ q = |\mathbf{V} - \mathbf{V}_p|,\ N = \rho_p/m_p,\ m_p = \frac{4}{3}\pi a^3\rho_p^0,\end{gathered} \tag{1.1}$$

where κ, c_V are the adiabatic index and the specific heat of the gas at constant volume; ρ_p°, c, density and the specific heat of the material of the particles; a, their radius; c_D, σ, drag coefficient of a sphere and heat-transfer coefficient for the surface, assumed to be known functions (see, e.g., [8]) of the Reynolds number Re, the Mach number M, and the Prandtl number Pr: $c_D = c_D(\mathrm{Re}, M)$, $\sigma = \sigma(\mathrm{Re}, \mathrm{Pr}, M)$. The functions appearing in Eq. (1.1) satisfy the boundary conditions obtained from the usual Rankine–Hugoniot relations, the condition of continuity of the functions V_p, ρ_p, T_p in the bow shock, and from the condition of impermeability of gas through the wing surface.

2. To solve this problem in the case of hypersonic flow over a thin wing at finite angle of attack, we use the thin shock-layer method [1], in which, as $\kappa \to 1$, $M_\infty \to \infty$ we represent all the desired functions in the strongly compressed gas layer between the windward surface of the wing and the bow shock wave in the form of expansions in the small parameter ε, equal to the ratio of densities at the shock

$$\varepsilon = \frac{\varkappa - 1}{\varkappa + 1}(1 + m^{-1}),\quad m = \frac{1}{2}(\varkappa - 1)\mathrm{M}_\infty^2\sin^2\alpha = O(1).$$

Here and below, the subscript ∞ denotes the parameters of the oncoming flow. We consider the most interesting and mathematically complex case of three-dimensional flow over a short wing, with a shock wave of unknown shape in the main approximation, assuming [7, 9], that for $\varepsilon \to 0$ the ratio of the semispan b to the root chord L is on the order $b/L = O(\varepsilon^{1/2}\tan\alpha)$, and that the relative thickness is $d = O(\varepsilon\tan\alpha)$. Let Oxyz be a rectangular coordinate system fixed in the wing. We introduce dimensionless variables on the order of one in the shock layer, refer all the dimensions along the axes x, y, and z, respectively, to L, $L\varepsilon\tan\alpha$, and $L\varepsilon^{1/2}\tan\alpha$, and retain the previous notation for the dimensionless variables. We assume that, in the incident flow, the velocity and temperature of the gas and the particles are the same. Then, estimating the order of the quantities [7], the desired functions can be written in the form of the following asymptotic expansions:

$$\begin{aligned}&u/V_\infty = u_0\cos\alpha + \varepsilon u_1\sin\alpha\,\mathrm{tg}\,\alpha + \ldots,\ v/V_\infty = \varepsilon v_1\sin\alpha + \ldots,\\ &w/V_\infty = \varepsilon^{1/2}w_1\sin\alpha + \ldots,\\ &(p - p_\infty)/(\rho_\infty V_\infty^2) = \sin^2\alpha\,(p_0 + \varepsilon p_1) + \ldots,\ q/V_\infty = q_0\cos\alpha + \ldots,\\ &\rho/\rho_\infty = \varepsilon^{-1}\rho_0 + \rho_1 + \ldots,\ T/T_\infty = (m + 1)T_0 + \ldots,\\ &u_p/V_\infty = u_{p0}\cos\alpha + \ldots,\ v_p/V_\infty = -v_{p0}\sin\alpha + \ldots,\\ &w_p/V_\infty = \varepsilon^{1/2}w_{p1}\sin\alpha + \ldots,\ N/N_\infty = N_0 + \ldots,\ T_p/T_\infty = T_{p0} + \ldots\end{aligned} \tag{2.1}$$

The functions appearing on the right side of the expansions are on the order of one for $\varepsilon \to 0$ and depend on the coordinates x, y, and z and a number of dimensionless similarity parameters. It can be seen from Eq. (2.1) that in the shock layer the velocity of the gas relative to the particles is hypersonic ($M \gg 1$). Taking into account that the Re is quite large ($Re \gg 1$), analogously to [4] we put c_D = const. Substituting Eq. (2.1) into Eq. (1.1) and the boundary conditions and going to the limit $\varepsilon \to 0$, assuming here that the following similarity parameters remain constant on the order of one, we find:

$$I = \frac{3c_D\tau_\infty}{8\varepsilon}\frac{L}{a}, \quad E = \frac{\sigma N_\infty}{c_V\rho_\infty}t_*,$$
$$I_p = \frac{3c_D\rho_\infty}{8\rho_p^0}\frac{L}{a}, \quad E_p = \frac{(m+1)\varepsilon\sigma}{cm_p}t_*,$$

where $t_* = L/V_\infty \cos\alpha$; the parameters I_p and E_p describe the action of the gas on the motion of the impurity particles, and the parameters I and E describe the inverse action of the impurity on the gas flow. The conditions assumed above, $I = O(1)$ and $E = O(1)$, correspond to the case where the presence of impurity in the zero-order approximation does not affect the gas flow and, as in a pure gas, $u_0 = p_0 = \rho_0 = T_0 = 1$.

Also, for particles in the zero-order approximation we obtain a system of ordinary differential equations containing only derivatives through the shock layer (the subscript 0 is omitted):

$$v_p\frac{du_p}{dy} = I_p q(u_p - 1), \quad \frac{dv_p}{dy} = I_p q, \quad \frac{d(Nv_p)}{dy} = 0,$$
$$v_p\frac{dT_p}{dy} = E_p\left(\frac{T_p}{m+1} - 1\right), \quad q = \left[(u_p - 1)^2 + v_p^2\,\mathrm{tg}^2\,\alpha\right]^{1/2}. \tag{2.2}$$

The boundary conditions at the shock give

$$u_{ps} = v_{ps} = N_s = T_{ps} = 1, \; y = S(x, z). \tag{2.3}$$

It follows from the first two equations of the system (2.2) that in the compressed layer $u_p - 1 = f(x, z)v_p$, whence, allowing for Eq. (2.3), we obtain

$$f(x, z) = 0, \; u_p = 1. \tag{2.4}$$

Further, from Eqs. (2.2) and (2.3) we find the form of the remaining functions

$$v_p = \exp\{I_{p\alpha}[y - S(x, z)]\}, \quad N = v_p^{-1},$$
$$T_p = 1 + m\left\{1 - \exp\left[-\frac{E_p}{(m+1)I_{p\alpha}}\left(\frac{1 - v_p}{v_p}\right)\right]\right\}, \quad I_{p\alpha} = I_p\,\mathrm{tg}\,\alpha. \tag{2.5}$$

The solution (2.4) and (2.5) obtained shows that the longitudinal velocity component of the particles in the shock layer is conserved, while the normal component is sharply reduced through the layer due to transfer of a certain portion of the momentum in that direction from particle to gas. This influence of the impurity on the gas shows up in the first approximation which describes the flow structure and the pressure distribution in the three-dimensional shock layer. The corresponding nonlinear system of equations and boundary conditions at the shock and the wind surface $y = B(x, z)$ have the form (we omit the subscript 1)

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad Du = 0, \quad Dw = 0,$$
$$Dv = -\frac{\partial p}{\partial y} - I_\alpha v_p, \quad I_\alpha = I\,\mathrm{tg}\,\alpha,$$
$$D(p - \rho) = \frac{2m}{m+1}I_\alpha v_p^2 + \frac{E}{v_p}\left(\frac{T_p}{m+1} - 1\right) \tag{2.6}$$
$$\left(D \equiv \frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right),$$
$$u_s = -\frac{\partial S}{\partial x}, \; v_s = \frac{\partial S}{\partial x} - \left(\frac{\partial S}{\partial z}\right)^2 - 1, \; w_s = -\frac{\partial S}{\partial z}, \; p_s = -2u_s - w_s^2 - 1,$$

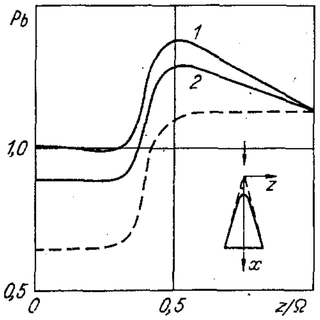

Fig. 1

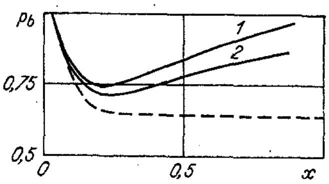

Fig. 2

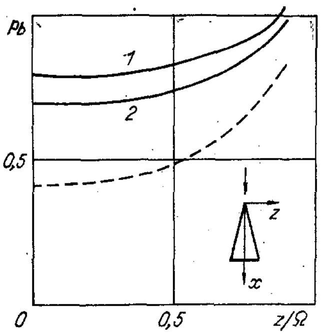

Fig. 3

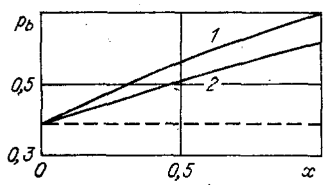

Fig. 4

$$\rho_s = 1 + p_s + \frac{m}{m+1}(2u_s + w_s^2), \quad y = S(x, z), \tag{2.6}$$

$$v_b = \frac{\partial B}{\partial x} + w_b \frac{\partial B}{\partial z}, \quad y = B(x, z).$$

From the first and third equations of this system, it follows that

$$D\frac{\partial w}{\partial y} = 0, \tag{2.7}$$

i.e., the basic property, established first in [9], of conservation of the flow component of vorticity along the streamlines remains valid, even in the presence of an impurity in the gas. This means that the kinematic flow picture given by Eqs. (2.6) and (2.7) and the corresponding boundary conditions analogously to [9] and the shape of the shock wave in this case are the same as in the pure gas [9]. This stems directly from the high gas density in the shock layer, because of which the above-mentioned momentum transfer from particle to gas influences the velocity field in the layer only in the higher approximations. However, the pressure increment due to this occurs also in the first approximation. In fact, denoting by p_c the known pressure in the pure gas, and taking account of Eq. (2.5), from the fourth equation of Eq. (2.6) we have

$$\frac{\partial p}{\partial y} = \frac{\partial p_c}{\partial y} + I_\alpha \exp\{I_{p\alpha}[y - S(x, z)]\}.$$

Integrating and satisfying the condition at the shock, we obtain the formula

$$p_b = p_{cb} + \frac{I}{I_p}[1 - \exp\{I_{p\alpha}[B(x, z) - S(x, z)]\}], \tag{2.8}$$

from which we can determine the pressure on the wing, allowing for the influence of the impurity, if we know the pressure and the shock shape in the pure gas. According to Eq. (2.8), the influence of the dust in the gas increases the pressure on the wing, and here the pressure increment depends directly on the parameter I and I_p and on the angle of attack, which appear in the expression for the relative velocity of the phases q. In the pure gas the dependence on α shows up only via the similarity parameter $\Omega = b/\varepsilon^{1/2}\tan\alpha$ [7]. Because of interphase energy transfer the Bernoulli integral for the equations of motion of the gas is invalid,

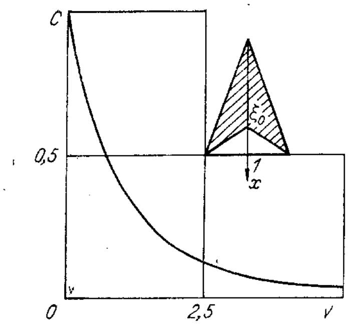

Fig. 5

and a first-order correction to the density must be applied from the last equation of Eq. (2.6).

By way of example, Figs. 1 and 2 show the pressure distributions along the span and along the root chord of a wing of hyperbolic shape in plan, washed by a flow with an attached shock at the leading edge. The solution, obtained in [10], for the case of flow of a pure gas over the wing, is shown by the broken line, and the lines 1 and 2 show the solution for a gas with impurity (α = 30 and 20°) for $I = I_p = 1$; the basic geometrical similarity parameter is $\Omega = 3.16$. It can be seen that the dustiness of the gas leads to an increased pressure and makes the distribution $p_b(z)$ nonmonotonic with a characteristic maximum roughly in the middle of the semispan. The pressure distribution $p_b(x)$ also becomes nonmonotonic and has a minimum at a large distance downstream of the wing apex.

The pressure distribution over the span of a wing with a detached shock wave is illustrated in Fig. 3, in the example of a planar triangular wing ($\Omega = 1.15$), where the broken line if the solution in the pure gas [11], and 1 and 2 refer to the gas with impurity (α = 40 and 30°) for $I = I_p = 1$. In this case its influence is to increase the pressure, but qualitatively the form of $p_b(z)$ does not change. In contrast with [11] the pressure along the root chord of this wing becomes monotonically increasing (Fig. 4). Investigations have shown that the impurity effect shows the same trends also in flow over a triangular wing of finite span at angles of attack close to $\pi/2$ [12].

An important characteristic of a body washed by a dusty gas is its accumulation efficiency C, equal to the ratio of the number of particles incident on the body per unit time to the number of particles that would fall on it in the absence of interphase interaction. To determine C one must calculate the particle trajectories, which is very difficult for three-dimensional motion. To obtain an upper estimate on C for the planar triangular wing we can use the solution in the vicinity of the plane of symmetry. The shock here has the form $S \approx \Delta_0 x$ (Δ_0 = const), and therefore the trajectory of particles entering the shock layer at $x = \xi$ has the form

$$y(x, \xi) = \frac{1}{I_{p\alpha}} \ln \frac{\Delta_0 I_p}{(1 + \Delta_0 I_p)\, e^{-\nu\xi} - e^{-\nu x}}, \quad \nu = \Delta_0 I_{p\alpha}.$$

We denote by ξ_0 the root of the equation $y(1, \xi_0) = 0$. Near the plane of symmetry all the particles arrive at the wing that enter the shock layer at $0 \le \xi \le \xi_0$. If we neglect the motion of particles along the span and assume that particles entering the shock layer within the shaded area (Fig. 5) reach the entire wing, then the estimate of the cumulative efficiency will be as follows:

$$C \approx \xi_0 = \frac{1}{\nu} \ln \frac{\nu \operatorname{ctg} \alpha + 1}{\nu \operatorname{ctg} \alpha + e^{-\nu}}.$$

For a wing with $\Omega = 1.15$ at $\alpha = 45°$ the dependence $C(\nu)$ is shown in Fig. 5. The limiting case $I_p \to 0$ corresponds to the gas having a negligibly small influence on the particles, because of which $C \to 1$; in the case $I_p \to \infty$, the flow of the mixture is equilibrium, and the particle velocity coincides with that of the gas flowing over the wing and, therefore, $C \to 0$.

We note that all the above results can be generalized easily to the case of unsteady flow of a dusty gas in a shock layer, if a wing with a surface shape variable with time is washed by a steady two-phase flow.

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ACOUSTIC RESONANCE IN SUBSONIC AERODYNAMIC INTERACTION OF CASCADES

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It is known [1] that acoustic resonance could take place in turbomachinery cascades when frequencies of any periodic disturbances coincide with characteristic frequencies of flow fluctuations in cascades. The results of studies on this phenomenon are given in [2, 3] for the case when acoustic disturbances are caused by fluctuations in flow in the trailing-edge wakes. However, the most powerful, constantly acting, and periodic source of disturbances in turbomachines is the aerodynamic interaction of the impeller and the guide vanes. The present work is devoted to the experimental and theoretical determination of conditions for its appearance.

1. Consider two annular cascades with one of them rotating about the axis of symmetry z at an angular velocity Ω. Introduce a stationary cylindrical coordinate system (·, θ, z) and also a moving system (r, θ_1, z) rigidly fixed to the rotating cascade so that

$$\theta = \theta_1 + \Omega t. \tag{1.1}$$

When the flow past each of these cascades is uniform, the velocities are periodic functions of θ with periods 2π/N, 2D/N_1, where N and N_1 are the number of blades in the stator and rotor cascades, respectively, i.e.,

$$V(r, \theta, z) = \sum_{n=-\infty}^{\infty} v_n(r, z) \exp(inN\theta),$$

$$V_1(r, \theta_1, z) = \sum_{n=-\infty}^{\infty} v_{1n}(r, z) \exp(inN_1\theta_1).$$

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